SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If $f(z)$ is analytic at z_0 with $f'(z_0) \neq 0$, prove that it maps a neighborhood of z_0 conformally and topologically onto a region.

17. If $f(z)$ is analytic is Ω, prove that $\int_\gamma f(z)dz = 0$ for every cycle γ which is homologous to zero in Ω.

18. Suppose that $u(z)$ is harmonic for $|z| < R$, continuous for $|z| \leq R$. Show that $u(a) = \frac{1}{2\pi} \int_{|z|=R} \frac{R^2 - |a|^2}{|z-a|^2} u(z)d\theta$ for all $|a| < R$.

19. State and prove Laurent theorem.

20. Explain Hadamard's theorem.

NOVEMBER/DECEMBER 2024

23PMA31 — COMPLEX ANALYSIS

Time : Three hours Maximum : 75 marks

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SECTION A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. Define the index of a point.

2. Compute $\int_{|z|=1} \frac{e^z}{z} dz$.

3. Define multiply connected regions.

4. State the Argument principle.

5. Define potential function.

6. State Poisson's formula.

7. Define Taylor's series.

8. Write the Maclawin expansion of e^z and $\sin z$.

9. Define Euler's constant.

10. State Jensen's formula.

SECTION B — (5 × 5 = 25 marks)

Answer ALL the questions.

11. (a) State and prove Cauchy's Integral formula.

Or

(b) Prove that an analytic function comes arbitrarily close to any complex value in every neighborhood of an essential singularity.

12. (a) Prove that a region Ω is simply connected iff $n(\gamma, a) = 0$ for all cycles γ in Ω and all points a which do not belong to Ω.

Or

(b) Let $f(x)$ be analytic except for isolated singularities a_j in a region Ω. Prove that $\frac{1}{2\pi i}\int_{\gamma} f(z)dz = \sum_{j} n(\gamma, a_j) \operatorname{Res}_{z=a_j} f(z)$ for any cycle γ which is homologous to zero in Ω and does not pass through any of the points a_j.

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13. (a) Compute $\int_0^{\pi} \frac{d\theta}{a + \cos\theta}$, $a > 1$.

Or

(b) If u_1 and u_2 are harmonic in a region Ω, prove that $\int_{\gamma} u_1 * du_2 - u_2 * du = 0$.

14. (a) Suppose that $f_n(z)$ is analytic in the region Ω_n and that the sequence $\{f_n(z)\}$ converges to a limit function $f(z)$ in a region Ω, uniformly on every compact subset of Ω prove that $f(z)$ is analytic in Ω.

Or

(b) State and prove Taylor's theorem.

15. (a) Prove that every function which is mesomorphic in the whole plane is the quotient of two entire function.

Or

(b) Derive the Poisson – Jensen formula.